

The change in crankcase volume  $A$  is the volume between the two piston displacements and is a function of  $d$  the phase angle between the pistons. Assuming identical cylinder bore diameters then

$$A(d) = \int_l^u (\sin x - \sin(x + d)) dx$$

where  $l$  and  $u$  are the two solutions of  $\sin x = \sin(x + d)$  when  $\sin(x + d) \geq \sin(x)$  over the range  $d$  ( $0 \leq d \leq \pi$ )

The solution is

$$A := \cos(a) - \cos(a + d) - \cos(b) + \cos(b + d)$$

where  $a = -\arctan(\sin(d)/(-1 + \cos(d)))$  and  $b = \pi - \arctan(\sin(d)/(-1 + \cos(d)))$  are the lower and upper limits that satisfy  $\sin(a + d) = \sin(a)$  and  $\sin(b + d) = \sin(b)$ .

This evaluates to

$$A(d) = 2 \frac{1}{\sqrt{1 + \frac{(\sin(d))^2}{(-1 + \cos(d))^2}}} - 2 \cos\left(-\arctan\left(\frac{\sin(d)}{-1 + \cos(d)}\right) + d\right)$$

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